RATIOS AND RATES

ACTIVITY 4.1 Investigative

RATIOS AND RATES

Activity Focus
• Writing ratios
• Equivalent ratios and proportions
• Rates, unit rates, and unit prices

Materials
• Measuring tape
• Clock with second hand or stopwatches
• Food coloring
• 2 clear cups (same size)
• Tablespoon
• Water
• BLM 3: Reading Passage (optional)

Chunking the Activity
Act. 1–6 #11–14 #29–30
#1–3 #15–16 #31–32
#4–6 #17–19 #33–37
#7–8 #20–23 #38
#9–10 #24–28

Paragraph Summarize/Paraphrase/Retell

Activities 1–6 Use Manipulatives, Look for a Pattern, Think/Pair/Share Have students work in pairs for these activities, which actively involve and motivate them. Do not introduce the term rate now. Later, students will apply what they learn about rates to find their personal rates.

Before starting, check that students can measure seconds with a clock or stopwatch. Let students choose a reading passage for Activity 2 or use the passage provided in Resources at the back of this Teacher’s Edition.

Activity 6 is most easily done in a hallway with 1-foot tiles, or you can use masking tape to mark off feet on the floor.

Some mathematics educators differ about the relationship of ratios and rates. Sometimes they are treated as distinct terms, so that one is not a special case of the other, and both are defined as comparisons.

Ratio: a comparison of numbers with the same units, such as 3 cups water: 5 cups juice
Rate: a comparison of two measurements with different units, such as $5 per 1 hour

Sometimes a rate is defined simply as a special type of ratio.

Ratio: any comparison of two numbers or measurements
Rate: a special ratio in which the two terms have different units

(Continued on next page)
Paragraph Marking the Text, Think Aloud, Vocabulary Organizer

Vocabulary Organizer
These questions serve as review of writing ratios. Students should understand that the order of the quantities, juice concentrate and water, determines the placement of the terms. Most sources give 2 to 3, 2:3, and \( \frac{2}{3} \) as the acceptable ways to write a ratio, but some sources accept \( 2 \div 3 \) as well. A writing math and mini-lesson are provided for students new to writing ratios.

Guess and Check
This question gives students an opportunity to think about ratio relationships before exploring them mathematically. Whatever the students answer here is acceptable at this point, as this is merely meant as an activator. In addition, this question brings forth the common misconception that if you add the same amount to both terms in a ratio, you will have an equivalent ratio.

Writing Math
Ratios can be written as fractions, with a colon (:), or using the word to. For example:
\( \frac{2}{3} \) or 8:9 or 8 to 9

The students must work together to plan the food and drinks. Ms. Yang already has some juice concentrate, napkins, and cups they can use. First the students need to figure out how to blend the concentrate with water to make enough juice for 26 people.

The label on the can of mix gives two mixing options.

Directions: Use one of these options
1. Add 2 cups concentrate to every 4 cups water, or
2. Add 3 cups concentrate to every 5 cups water.

You can compare two quantities, or show a relationship between them, by writing a ratio. The numbers that are compared are called terms.

1. The directions for mixing the juice show a relationship between what 2 quantities? concentrate and water
2. Write a ratio in 3 different ways to show this relationship for each option.

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cups concentrate</td>
<td>3 cups concentrate</td>
</tr>
<tr>
<td>4 cups water</td>
<td>5 cups water</td>
</tr>
<tr>
<td>2 \div 4</td>
<td>3 \div 5</td>
</tr>
<tr>
<td>2:4</td>
<td>3:5</td>
</tr>
<tr>
<td>2 to 4</td>
<td>3 to 5</td>
</tr>
</tbody>
</table>

3. The students agree that they want to make juice with the most flavor. Which option do you think they will choose? Explain why you think they will make that choice.

☐ Option 1 is more flavorful.
☐ Option 2 is more flavorful.
☐ Options 1 and 2 are equally flavorful.

Teacher to Teacher (Continued)
Consider the following situation:

• Would 1 ounce of food coloring per pound of dough be a rate because it compares ounces to pounds, while 1 ounce of food coloring per 16 ounces of dough would be a ratio since it relates ounces to ounces?
• Are interest rates truly rates given that they technically compare similar units, money and money?

Most mathematicians agree that ratio is a more general term that encompasses rates and that ratio is simply a relationship between two quantities. In this activity, ratio is used as a generic term while rate applies to ratios that involve two different kinds of units.
MINI-LESSON: Writing Ratios

Ratios can be written in three different ways. If there are 3 red marbles and 4 blue marbles, we say the ratio of red marbles to blue marbles is 3 to 4 (3 red marbles to every 4 blue marbles). This ratio can also be written as 3:4 or $\frac{3}{4}$. The first number must represent the first thing listed, in this case the red marbles. Write a ratio to represent each of the following comparisons.

1. A class has 14 boys and 11 girls
2. 24 cookies for every 1 batch
3. A bag holds 2 pens, 4 pencils, and 1 notebook. What is the ratio of notebooks to pens?

ACTIVITY 4.1 Continued

4. Create Representations You can make up a “juice concentrate” by adding food coloring to water. Then dilute that mixture with water to simulate the 2:4 and 3:5 ratios in the clear containers. You may need to experiment a bit to have a noticeable difference in the two solutions. You want the students to be able to see that the ratios are not equivalent, so the cup showing 2:4 should be lighter than the cup showing 3:5.

5. Quickwrite You may want to continue to add the “concentrate” to the cups of water, showing different ratios and why just adding 1 part of concentrate and 1 cup of water, or 2 parts of concentrate and 2 cups of water, and so on, does not keep the same ratio of concentrate to water.

6. Quickwrite, Debriefing It is important for students to understand that ratios have a multiplicative relationship, and not an additive one. Give various examples to solidify this before moving on if needed.
7-8  Create Representations (7a), Look for a Pattern (7b, c, 8), Quickwrite (7c, 8b), Think/Pair/Share (8) These questions give students a chance to explore (in tabular form) what happens when the terms of ratios are doubled, tripled, and so on. A connection should be made to tables of values for functions, with which the students should be familiar. By completing these tables they are creating equivalent ratios.

7. Using ratio tables is a way to compare the ratios $\frac{2}{4}$ and $\frac{3}{5}$.
   a. Complete each ratio table to show the relationship of juice concentrate to water for each mixing option if you double the recipe, triple it, and so on.

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Juice concentrate</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Option 2</th>
<th>Juice concentrate</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

b. Highlight the column in each table that has the same number of cups of juice concentrate and write each ratio below.
   Option 1: 6 to 12, Option 2: 6 to 10

c. What does this tell you about the strength of each mixture?
   Answers may vary. Sample answer: Because the two mixtures have the same amount of juice concentrate, the one with more water is going to be less flavorful. So Option 1 is less flavorful.

8. Now use a different color to highlight the column in each table that has the same number of cups of water.
   a. Write each ratio.
      Option 1: 10 to 20, Option 2: 12 to 20

b. What does this tell you about the strength of each mixture?
   Answers may vary. Sample answer: The two mixtures have the same amount of water, so the one with more juice concentrate will be more flavorful. Option 2 is more flavorful.
9. If the students increase the juice concentrate 1 cup at a time, find how much water they would need to use in each case.

a. Complete the ratio tables below.

<table>
<thead>
<tr>
<th>Option</th>
<th>Concentrate (m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water (w)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Option</td>
<td>Concentrate (m)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Water (w)</td>
<td>1  1 2</td>
<td>3  1 3</td>
<td>5</td>
<td>6  2 1</td>
<td>3</td>
</tr>
</tbody>
</table>

b. What patterns do you notice in the table for Option 1? Answers will vary. Every time one cup of mix is added, two cups of water are added.

c. What is the rule for the Option 1 table? $m = \frac{1}{2}w$ or $w = 2m$

d. What patterns do you notice in the table for Option 2? Answers will vary. For every cup of mix, you add $\frac{1}{2} \frac{2}{3}$ cups of water.

e. What is the rule for the Option 2 table? $m = \frac{3}{5}w$ or $w = \frac{5}{3}m$

Relationships that have the same ratio are called proportional relationships and can be represented with the algebraic rule $y = mx$. The amount, $m$, is the factor by which $y$ increases each time. It represents a constant rate of change.

10. Graph the data and compare the graphs.

a. List the ratio pairs from each ratio table in Question 9 as ordered pairs. Option 1: (1, 2), (2, 4), (3, 6), (4, 8), (5, 10); Option 2: points $(1, \frac{1}{2}), (2, \frac{3}{3}), (3, 5), (4, \frac{6}{3}), (5, \frac{8}{3})$

b. Graph the ordered pairs on the grids in the My Notes space.

c. What do you notice about the graph of the data in each ratio table? Answers may vary. Sample answer: The points on each graph seem to be in a straight line.
**ACTIVITY 4.1 Continued**

11-12 **Think/Pair/Share**

Question 11 emphasizes the part-to-part relationship of this ratio, forcing students to consider the whole, which is 8 cups of juice. Realistically, this is not enough juice for the class, as they need enough juice for 26 people. In Question 12, the students are led through doubling the recipe, or terms of the ratio. This problem is intentionally not written as equivalent fractions in order to stress the proportional reasoning of doubling each term.

13-14 **Think/Pair/Share (14), Create Representations (13c), Vocabulary Organizer (14)** As the 6:10 ratio only makes 16 cups of juice, the students now discover in Question 13 that they will need to multiply each term of the ratio by 7. \(\frac{3}{5} \times 7 = \frac{21}{35}\), which makes a total of 56 cups. By setting \(\frac{3}{5}\) equal to \(\frac{21}{35}\) they have written a proportion in Question 14.

11. Ms. Yang’s class decides to make the juice using the concentrate and water from Option 2.
   - **a.** Explain how to determine the number of cups of juice one batch of this mixture will make. Add the amount of concentrate and the amount of water. \(3 + 5 = 8\) cups
   - **b.** To give each of the 26 people attending Math Night 2 cups of juice, how many cups do they need to make? Explain how you determined your answer. \(52. I\ \text{multiplied}\ 26\ \text{by}\ 2.\)

12. To make more juice, the students double their recipe.
   - **a.** Express the ratio of concentrate to water when they revise the recipe. \(6\ \text{cups mix}\)
   - **b.** How many cups of juice will that make? Explain why this is or is not enough juice for the party. \(16\ \text{cups};\ \text{it}\ \text{is}\ \text{not}\ \text{enough}\ \text{because}\ \text{the}\ \text{students}\ \text{need}\ \text{52}\ \text{cups},\ \text{and}\ \text{doubling}\ \text{the}\ \text{recipe}\ \text{gives}\ \text{only}\ \text{16}\ \text{cups}.\)

13. The students need to increase their mixture to make enough juice to serve two cups each to the 26 people attending.
   - **a.** Find a ratio equal to 3 cups concentrate per 5 cups water that provides enough juice, with as little extra juice as possible. Explain how you determined your answer. \(\frac{21}{35}\)
   - **b.** Write a number sentence that shows the original ratio is equal to your new ratio. \(\frac{3}{5} = \frac{21}{35}\)

The equation you just wrote shows a proportional relationship, and is called a **proportion**.

14. By writing an equivalent ratio, the students find out how to make more juice while keeping the same relationship between mix and water.
   - **a.** How many cups of juice will the students make using the ratio you found in Question 13b? \(56\)
   - **b.** How many extra cups of juice will there be? \(4\)

**MATH TERMS**

A **proportion** is a mathematical statement describing two ratios that are equal to each other.
Now the students must decide what kind of pizza to order. They decided on pepperoni and extra cheese. They found that 16 people want pepperoni and 10 people want extra cheese.

15. Write a ratio in fraction form that shows the relationship of pepperoni slices to cheese slices.

16. The students found that the average number of slices each person would eat was 2 slices. Write a ratio equivalent to the one you wrote for Question 15 that shows the relationship of pepperoni to cheese slices, assuming each person will eat 2 slices.

17. Use this ratio to determine the total number of pizza slices they need. Show your work.

18. Another way to figure out the total slices needed is to write a ratio comparing pizza slices to people. Write the average number of slices per 1 person as a ratio in fraction form.

What you have just written is a special type of ratio known as a rate. This rate shows a relationship between quantities measured with different units (slices and people). Earlier, when pepperoni slices were compared to cheese slices, you compared different toppings (pepperoni and cheese), which had the same unit (slices). This type of ratio is also a rate.

When the rate is per 1 unit, such as slices per 1 student, it is called a unit rate. Unit rates are easy to spot because they are often written with the word per or with a slash (/) (for example, slices per person or slices/person).

19. Name at least 2 other situations where you have noticed a relationship expressed with the word per.

Answers may vary. Sample answers: miles per hour, heart beats per minute, $10 per hour, 10 cents per minute, $3.15 per gallon.

20. Think/Pair/Share This question gives students more practice with writing ratios. Students should be encouraged to include labels so that they can see they are comparing similar units, slices to slices, as well as to emphasize its property as a ratio, versus the fractional property of part to whole: 16 pepperoni slices/10 cheese slices. While some ratios have fractional meaning, there is no part to whole meaning shown by pepperoni to cheese slices. However, it is important that students are comfortable using the fraction form for ratios, as it makes computation with ratios easier.

21. Think/Pair/Share In Question 16, students learned that Ms. Yang’s class will eat an average of 2 slices per person. In this question they write this in mathematical terms as a rate: 2 slices/1 student. This helps them make a connection with prior knowledge.

Paragraph Marking the Text

Students are moving from their understanding of ratios to learning about rates and unit rates as they make decisions about purchasing pizza for the party. Until now, they were relating quantities with the same units, such as cups concentrate and cups water, while now they will relate quantities with different units.

Students learn to use equivalent fractions, unit rates, and proportions to solve missing value problems and numerical comparison problems.

Paragraph Marking the Text

Students connect this new terminology to real-life instances where they have used rates.
ACTIVITY 4.1 Continued

20-21 Activating Prior Knowledge (21) Students see how to set up a proportion in order to solve for a missing value. Although solving for the missing value as if they were finding equivalent fractions is a procedure students have used before, they may need review. Guide students to remember that to write equivalent fractions, they multiply the given fraction by 1 in any fractional form. This is different conceptually than telling students to multiply the numerator and denominator by the same number. Remind students that multiplying by 1 does not change the value of a number, because of the multiplicative identity.

22 Identify a Subtask This question returns students to the real-life context and reviews how to interpret remainders in division. Students reason that an answer of 6 R 4 means the class will eat 6 whole pizzas (48 slices), and 4 slices of another pizza. They must buy 7 pizzas in order to have enough pizza.

23 Think/Pair/Share This question gives students more practice with the equivalent fraction strategy, and introduces them to working backward. Instead of multiplying by 1 to find a missing value in an equivalent ratio, they divide by 1 to find unit rate. Using proportions here introduces students to simplifying ratios.

24 Students learn how to find the unit rate. They may reason that if 2 pizzas cost $19.98, then 1 pizza costs half that.

20. Use the unit rate to find the total number of slices needed. Set up a proportion. Fill in the values you know.

<table>
<thead>
<tr>
<th>Unit Rate</th>
<th>Rate for Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slices/Person</td>
<td>Total Slices/Total People</td>
</tr>
<tr>
<td>2 slices</td>
<td>? slices</td>
</tr>
<tr>
<td>1 person</td>
<td>26 people</td>
</tr>
</tbody>
</table>

21. Finding equivalent rates is just like finding equivalent fractions. Rewrite the proportion and use the Property of One to solve. Think of this as finding an equivalent fraction.

22. If a large pizza is cut into 8 slices, how many pizzas must the students buy? 7 pizzas

Two pizza places have free delivery to the school.

- **Mama T’s Pizza**
  - Get 2 large 1-topping pizzas for just $19.98!
  - 1 large 1-topping pizza is $10.99

- **Toni’s Pizza**
  - 1 large 1-topping pizza for $10.99
  - 3 large 1-topping pizzas: only $29.70

23. Is $19.98 for 2 pizzas at Mama T’s Pizza a good deal? To determine this, you must find the price per pizza. Find the price per pizza by dividing the unit dollars (or price) by the unit pizzas to get "dollars/pizzas" or "price per pizza".

\[
\frac{19.98}{2 \text{ pizzas}} = \frac{9.99}{1 \text{ pizza}}
\]

Notice, by setting up the ratio \( \frac{19.98}{2 \text{ pizzas}} \), you treat it as a fraction and divide 19.98 by 2. You also divide the unit dollars (or price) by the unit pizzas to get "dollars/pizzas" or "price per pizza".
### ACTIVITY 4.1 Continued

#### 25. Another way to find the cost of each pizza is to reason this way: “If $19.98 is the cost of 2 pizzas, then how many dollars does 1 pizza cost?”

\[
\begin{array}{c}
\$19.98 \\
2 \text{ pizzas}
\end{array} \rightarrow \begin{array}{c}
\$9.99 \\
1 \text{ pizza}
\end{array}
\]

For this way of reasoning, you think about *how much for one.* Think of this as finding the *unit rate.* When a problem involves working with money, the unit rate is called the **unit price.**

#### 26. How much do the students save by using this deal instead of buying 2 pizzas at regular price?

- $1.00 per pizza, or $2 for 2 pizzas

#### 27. What is the price per pizza for the deal at *Toni’s Pizza?*

- $9.90

#### 28. To decide where they will get the better deal, the students cannot simply compare rates. They need a specific number of pizzas, so the better deal may depend on how many pizzas they are buying.

- **a.** Determine how much it would cost to buy 7 pizzas from *Mama T’s Pizza.* The students can use the deal for every 2 pizzas they buy, but the seventh pizza will be at regular price. Show how to use a proportion to help determine your answer.

  \[
  \frac{19.98}{2 \text{ pizzas}} \times \frac{3}{3} = \frac{59.94}{6 \text{ pizzas}}; \quad 59.94 + 10.99 = 70.93
  \]

- **b.** Determine how much it would cost to buy 7 pizzas from *Toni’s Pizza.* Show your work.

  \[
  \frac{29.70}{3 \text{ pizzas}} \times \frac{2}{2} = \frac{59.40}{6 \text{ pizzas}}; \quad 59.40 + 10.99 = 70.39
  \]

- **c.** Where should the students buy their pizza? Explain. Explanations may vary. Sample answer: *Toni’s Pizza;* $70.93 − $70.39 = $0.54 cheaper.

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**Identify a Subtask (25)** Question 25 relates unit rate to the context. Students may solve this using the unit rate or by finding the price of 2 pizzas at regular price.

**Question the Text, Create Representations (a, b), Quickwrite (c), Group Presentation** This problem brings up the issue of “better deal.” This is a big discussion point when discussing smart shopping.

The group of students in the problem must understand that the unit price only tells them how good the deal is, but that it cannot be applied to their total number of pizzas. For instance, if the students buy 3 pizzas from Mama T’s Pizza, they only get 2 pizzas with the unit price of $9.99 (the 2-for deal), but the third pizza will cost the regular price, $10.99. Although Toni’s Pizza has the better deal when considering unit price, it is not necessarily the better deal for these students who need 7 pizzas. Guide the class to look at various possibilities for buying the pizza. If the students only needed to buy 4 pizzas, it would be smarter to use the deal of $19.98 for 2 pizzas, as their total would be $19.98 \times 2 = $39.96. Using the Toni’s Pizza deal for $29.70 for 3 pizzas + 1 pizza at regular price would cost a total of: $29.70 + 10.99 = $40.69.

**Suggested Assignment**

CHECK YOUR UNDERSTANDING

- p. 194, #3b–5

UNIT 4 PRACTICE

- p. 235, #3–5
The next series of questions moves students from using the fraction strategy and factor-of-change strategy to find total pizza cost, to using the unit rate strategy to compare rates in numerical comparison problems as they plan to purchase paper plates. This time “better deal” is based upon unit rate, as they are not purchasing exact amounts.

Students should answer Question 29a and 29b using only logic, without doing any written work. Since Part c cannot be done in the same way, it gives students a need to use unit rates.

Look for a Pattern, Quickwrite (b), Think/Pair/Share, Self Revision/Peer Revision This problem guides students through using unit rates to solve numerical comparison problems. After writing each rate in fractional form students see that they cannot easily multiply the terms of one ratio to have common terms with the other. Thus, they must find unit prices.

Now that the juice and pizza are figured out, the students must purchase paper plates. They are not concerned about buying a specific number of each this time, because they will use the extra plates in the future. They are searching for the best deals.

29. The table shows rates for the cost of paper plates at three different stores. Each store has two options.

<table>
<thead>
<tr>
<th>Paper Supplies</th>
<th>Party!</th>
<th>Local Grocer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.99/100 plates</td>
<td>$3.39/15 plates</td>
<td>$3.39/15 plates</td>
</tr>
<tr>
<td>$6.90/100 plates</td>
<td>$3.39/25 plates</td>
<td>$6.39/24 plates</td>
</tr>
</tbody>
</table>

a. Decide which is the better rate at Paper Supplies. Explain your thinking.
   Answers may vary. Sample answer: Both rates at Paper Supplies have the same number of plates, but that the first is more expensive. So $6.90/100 plates is the better deal.

b. Decide which is the better rate at Party! Explain your thinking.
   Answers may vary. Sample answer: Both rates at Party! have the same cost, but $3.39/25 plates gives more plates and therefore is the better deal.

c. Can you easily figure out the better rate at Local Grocer? Why or why not?
   Explanations may vary. Sample answer: No, neither the price nor the number of plates is the same, and while one has a greater number of plates, the cost is also higher, so you cannot use simple reasoning.

30. Now consider only the rates at Local Grocer.

a. Write each rate at Local Grocer in fractional form.
   $3.39/15 plates and $6.39/24 plates

b. Can you multiply 15 plates by something to get 24, or $3.39 by something to get $6.39? How does this affect your ability to compare these two rates as you did for the Paper Supplies and Party! stores?
   Answers may vary. Sample answer: There is no whole number by which I can multiply $3.39 to get $6.39 or 15 to get 24.
c. When it is not easy to find an equivalent fraction to compare quantities, find the unit rate for each deal to find the unit price (price per plate).

\[
\frac{3.39}{15 \text{ plates}} = \frac{0.226}{1 \text{ plate}}
\]

\[\$0.23\] per plate or \[\$0.23/\text{plate}\]

d. Use this unit price (price per plate) to find the cost for 24 plates. Is that more or less than the other rate of \$6.39/24 plates?

\[
\frac{0.23}{1 \text{ plate}} = \frac{5.52}{24 \text{ plate}}
\]

\$5.52/24 plates is less than \$6.39/24 plates.

31. Another way to find the cost of a pack of 24 plates that has the same rate as \$3.39/15 plates is to write a proportion. Let \(c\) represent the unknown cost of the 24 plates.

\[
\frac{3.39}{15 \text{ plates}} = \frac{c}{24 \text{ plates}}
\]

To figure out a rule you can use to solve for \(c\), think about some procedures you already know. Finish solving the equations for \(c\), but do not simplify the terms. Explain what steps you use.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solve for (c)</th>
<th>Do not simplify.</th>
<th>What are the steps?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\frac{3}{5} = \frac{c}{2})</td>
<td>(5 \times 3 = c)</td>
<td>(5 \times 3 = c)</td>
<td>Multiply both sides by 5.</td>
</tr>
<tr>
<td>b. (\frac{3}{7} = 4)</td>
<td>(c = 7 \times 4)</td>
<td>(c = 7 \times 4)</td>
<td>Multiply both sides by 7.</td>
</tr>
<tr>
<td>c. (\frac{3}{5} = \frac{c}{9})</td>
<td>(\frac{3 \times 9}{5} = c)</td>
<td>(\frac{3 \times 9}{5} = c)</td>
<td>Multiply both sides by 9.</td>
</tr>
<tr>
<td>d. (\frac{c}{4} = \frac{2}{7})</td>
<td>(c = \frac{4 \times 2}{7})</td>
<td>(c = \frac{4 \times 2}{7})</td>
<td>Multiply both sides by 4.</td>
</tr>
</tbody>
</table>

e. What are the similarities in Parts a–d?

Answers may vary. Sample answer: I always multiplied both sides of the equation by denominator of the term containing the variable.
Paragraph Look for a Pattern, Discussion Group Students study three worked-out solutions to proportions and find that they can multiply cross terms, and then divide by the other term, which is diagonally across from the a. This is sometimes called: cross multiply and divide.

Look for a Pattern (c), Quickwrite (c, d), Discussion Group Encourage discussion of the solutions so that you can determine whether students understand and can apply the algorithm.

Quickwrite, Discussion Group This problem allows students to simplify the terms to find the total cost of the 24 plates. They find that their answer is not the same as when they used a unit rate in Question 30d, because in Question 30d students rounded 0.226 to $0.23, and then multiplied it by 24. Using the algorithm in this question, students did not round. If students are unable to determine this reason, use guided questioning to help them figure it out, but do not tell them the reason. By having students go back to Question 30d and use $0.226 \times 24$ they will see that their answer now matches this question. They should not change their answer to Question 30d. Explain that they have compared the answers to verify that the processes are the same, and it is only rounding that may make the final answers different.

MINI-LESSON: Understanding the Algorithm

If students struggle with seeing patterns as they undo operations, help them to see relationships using equivalent fractions, similar to the algorithm for comparing fractions discovered in Unit 1.

To solve for c:

\[
\frac{3.39}{15} = \frac{c}{24}
\]

\[
\frac{3.39 \times 24}{360} = \frac{c \times 15}{360}
\]

Consider only the numerators now that the denominators are common:

\[
3.39 \times 24 = c \times 15, \text{so } \frac{3.39 \times 24}{15} = c
\]

This process shown in solving these proportions is the cross products algorithm for solving proportions.

33. In question 31, you wanted to solve \( \frac{3.39}{15 \text{ plates}} = \frac{c}{24 \text{ plates}} \).

a. Use the cross products algorithm to find the cost of 24 plates. $5.42

b. How does this compare to the answer when you used the unit price in Question 30d?

Answers may vary. Sample answer: With the algorithm I got $5.42 and using the unit price I got $5.52.
ACTIVITY 4.1 Continued

34. The second rate at Local Grocer is $6.39/24 plates.
   a. Find the unit price for the rate of $6.39/24 plates.
      \[ \frac{0.26625}{1} \approx 0.27 \]
   b. Which is the better rate at Local Grocer? Explain.
      Answers may vary. Sample answer: $3.39/15 plates is the better deal; $0.23 is less than $0.27.

35. The students decide to buy the packages of plates priced $3.39/15 plates. They set up the proportion \( \frac{3.39}{15} = \frac{c}{30} \).
   a. What do the numbers and the variable in the proportion represent?
      $3.39$ is the cost of 15 plates; \( c \) is the cost of 30 plates.
   b. Solve the proportion for \( c \).
      \[ c = 6.78 \]

36. The students want to know what a pack of 26 plates would cost if the unit price were the same as 15 plates for $3.39.
   a. Use the unit price to find the cost of a pack of 26 plates at that rate.
      \[
      \frac{3.39}{15 \text{ plates}} = \frac{0.23}{1 \text{ plate}} = \frac{5.98}{26 \text{ plates}}
      \]
   b. Write a proportion to find the cost of a pack of 26 plates at that rate. Let \( c \) represent the cost.
      \[ \frac{3.39}{15} = \frac{c}{26} \]
   c. Use the algorithm to determine the total cost.
      \[ c = 5.876, \text{ about } $5.88 \]
   d. Did you get the same answers for Parts a and c? Explain.
      Answers may vary. Sample answer: No; I rounded 0.226 to 0.23 in Part a so I got different answers.

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The answer to Question 36d will vary depending on whether the students used a rounded value to answer Question 36a. If students did not round, both methods give the same answer.
ACTIVITY 4.1 Continued

Think/Pair/Share This question brings the pieces of the activity back together. They have now decided on the best deals for each purchase, and must find the cost per person.

Activities 1–6 Create Representations (1–6), Group Presentation (1–6) Students apply all the strategies they have learned for working with rates in order to find their personal rates for the activities they sampled at the start of the unit. They should use the data they collected during the introduction.

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Quickwrite, Discussion Group, Create Representations, Think/Pair/Share, Group Presentation

37. The students figure the party is going to cost $77.71. If all 26 people are going to share the cost equally, how much do they each need to contribute?

Now that you know about rates and unit rates, use what you have learned and your results from the first page of activities to find your personal rates for each of the tasks!

38. For each activity, use your data to set up a proportion. Next use one of the three methods you have learned to solve the proportion. Then enter your rate for that activity. As you do this, use each method at least once.

Activity 1: Writing Speed

a. How many times can you write math in 1 minute with your dominant hand?
   • Set up a proportion:
   • Solve it:
   • Express your result using a complete sentence.

b. What is your rate per minute with your non-dominant hand?

c. About how many times faster are you with your dominant hand?

Activity 2: Reading Speed

If there are about 250 words on a page, how many pages could you read in an hour based on the data you collected earlier?
Activity 3: Heart Rate
Heart rate is calculated in beats per minute. Express your heart rate in beats per minute.

Activity 4: Jumping Jacks
How many jumping jacks can you do in 90 seconds?

Activity 5: Are You Tongue Twisted?
About how many times can you say, "Peter Piper picked a peck of pickled peppers" in a minute?

Activity 6: Are You a Fast Walker?
How many feet can you walk in an hour? How many miles per hour can you walk? (There are 5,280 ft in a mile.)

Summary: How Do You Solve Proportions?
Look back at the ways you used to find the rates for the six activities. Find the method you used most often and explain why you chose this method.

As a possible follow-up activity, allow students to collect class data on what kind of pizza they eat and how many slices. If possible, students may use this data to plan their own class party.

Suggested Assignment
CHECK YOUR UNDERSTANDING
p. 194, #6–8
UNIT 4 PRACTICE
p. 235, #6–8

Connect to AP
Including units when computing provides an additional layer of sense-making when solving problems. For example, when finding a total distance traveled, the result should be units of length.

How far did Bob travel in two hours if he averaged 60 miles per hour?

The units in this result make sense.

\[
\frac{60 \text{ mi}}{\text{hr}} \cdot 2 \text{ hr} = 120 \text{ mi}
\]

The units in this result do not.

\[
\frac{60 \text{ mi}}{\text{hr}} \div 2 \text{ hr} = 30 \frac{\text{mi}}{\text{hr}}
\]

In AP mathematics, it is always important to pay attention to the units in a problem. Leaving units off an answer or giving incorrect units in an answer can cost students a valuable point on an AP examination.
CHECK YOUR UNDERSTANDING

1. Write a ratio in three different ways to represent the number of boys to the number of girls in the class.

<table>
<thead>
<tr>
<th>Girls</th>
<th>Boys</th>
</tr>
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<tbody>
<tr>
<td>12</td>
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2. Write a ratio for each situation.
   a. 310 heartbeats per 5 minutes
   b. $68 for 8 hours of work
   c. Work 40 hours in 5 days

3. A recent study shows that out of 100 pieces of a popular multicolored snack, there will usually be the following number of pieces of each color.

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   a. The numbers for two colors form a ratio that is equal to \( \frac{7}{12} \). What are the colors? What is their ratio?
   b. If there were 500 pieces, about how many would be red?

4. Kate made lemonade using a powder mix. She used 4 scoops of mix to a gallon of water.
   a. Write a proportion to determine the amount of water to mix with 12 scoops of mix.
   b. If Kate mixes less lemonade powder with more water, how will her mixture be affected?

5. Jaden travels 520 miles in 8 hours.
   a. Use a proportion to find his average rate per hour.
   b. Show why the formula \( d = rt \) is actually a rate problem.

6. It is about 2508 miles from Orange County in California to Orange County in Florida. With an average speed of 70 miles per hour, about how long will it take to drive from one to the other?

7. Which is the better cell phone deal if you consider only cost per minute: 450 minutes for $69.99 or 900 minutes for $89.99? Show how you know.

8. MATHEMATICAL REFLECTION
   Why does the cross-products method work when solving a proportion? Use an example to explain your reasoning.

Write your answers on notebook paper. Show your work.

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