# Algebra II Vocabulary Cards
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Natural Numbers

The set of numbers
1, 2, 3, 4...

Real Numbers

Rational Numbers
Integers
Whole Numbers

Natural Numbers

Irrational Numbers
Whole Numbers

The set of numbers 0, 1, 2, 3, 4...

Real Numbers

Rational Numbers

Integers

Whole Numbers

Natural Numbers

Irrational Numbers
Integers

The set of numbers
...-3, -2, -1, 0, 1, 2, 3...
Rational Numbers

The set of all numbers that can be written as the ratio of two integers with a non-zero denominator

\[ \frac{3}{5}, \quad -5, \quad 0.3, \quad \sqrt{16}, \quad \frac{13}{7} \]
Irrational Numbers

The set of all numbers that cannot be expressed as the ratio of integers

\[ \sqrt{7}, \pi, -0.23223222322223\ldots \]
Real Numbers

The set of all rational and irrational numbers
Complex Numbers

The set of all real and imaginary numbers
Complex Number

\[ a \pm bi \]

\( a \) and \( b \) are real numbers and \( i = \sqrt{-1} \)

A complex number consists of both real \((a)\) and imaginary \((bi)\) but either part can be 0

<table>
<thead>
<tr>
<th>Case</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 0 )</td>
<td>0.01i, -i, ( \frac{2i}{5} )</td>
</tr>
<tr>
<td>( b = 0 )</td>
<td>( \sqrt{5}, 4, -12.8 )</td>
</tr>
<tr>
<td>( a \neq 0, b \neq 0 )</td>
<td>39 – 6i, -2 + ( \pi i )</td>
</tr>
</tbody>
</table>
Absolute Value

\[ |5| = 5 \quad | -5 | = 5 \]

The distance between a number and zero

5 units

5 units
# Order of Operations

| Grouping Symbols | ( )  
|                 | { }  
|                 | [ ]  
|                 | |absolute value|  
|                 | fraction bar |
| Exponents       | $a^n$ |
| Multiplication  | Left to Right |
| Division        | Left to Right |
| Addition        | |
| Subtraction     | |
Expression

\[ x \]

\[-\sqrt{26} \]

\[ 3^4 + 2m \]

\[ 3(y + 3.9)^2 - \frac{8}{9} \]
Variable

\[2(y + \sqrt{3})\]

\[9 + x = 2.08\]

\[d = 7c - 5\]

\[A = \pi r^2\]
Term

\[3x + 2y - 8\]

3 terms

\[-5x^2 - x\]

2 terms

\[\frac{2}{3}ab\]

1 term
Scientific Notation

\[ a \times 10^n \]

1 ≤ |a| < 10 and n is an integer

Examples:

<table>
<thead>
<tr>
<th>Standard Notation</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>17,500,000</td>
<td>1.75 \times 10^7</td>
</tr>
<tr>
<td>-84,623</td>
<td>-8.4623 \times 10^4</td>
</tr>
<tr>
<td>0.0000026</td>
<td>2.6 \times 10^{-6}</td>
</tr>
<tr>
<td>-0.080029</td>
<td>-8.0029 \times 10^{-2}</td>
</tr>
</tbody>
</table>
Exponential Form

\[ a^n = a \cdot a \cdot a \cdot a \ldots, \quad a \neq 0 \]

Examples:

\[ 2 \cdot 2 \cdot 2 = 2^3 = 8 \]

\[ n \cdot n \cdot n \cdot n = n^4 \]

\[ 3 \cdot 3 \cdot 3 \cdot x \cdot x = 3^3 x^2 = 27x^2 \]
Negative Exponent

\[ a^{-n} = \frac{1}{a^n}, \quad a \neq 0 \]

Examples:

\[ 4^{-2} = \frac{1}{4^2} = \frac{1}{16} \]

\[ \frac{x^4}{y^{-2}} = \frac{x^4}{\frac{1}{y^2}} = \frac{x^4}{\frac{1}{y^2}} \cdot \frac{y^2}{y^2} = x^4 y^2 \]

\[ (2 - a)^{-2} = \frac{1}{(2 - a)^2}, \quad a \neq 2 \]
Zero Exponent

\[ a^0 = 1, \quad a \neq 0 \]

Examples:

\[ (-5)^0 = 1 \]
\[ (3x + 2)^0 = 1 \]
\[ (x^2 y^{-5} z^8)^0 = 1 \]
\[ 4m^0 = 4 \cdot 1 = 4 \]
Product of Powers Property

\[ a^m \cdot a^n = a^{m+n} \]

Examples:

\[ x^4 \cdot x^2 = x^{4+2} = x^6 \]

\[ a^3 \cdot a = a^{3+1} = a^4 \]

\[ w^7 \cdot w^{-4} = w^{7+(-4)} = w^3 \]
Power of a Power Property

\[(a^m)^n = a^{m \cdot n}\]

Examples:

\[(y^4)^2 = y^{4 \cdot 2} = y^8\]

\[(g^2)^{-3} = g^{2 \cdot (-3)} = g^{-6} = \frac{1}{g^6}\]
Power of a Product Property

\[(ab)^m = a^m \cdot b^m\]

Examples:

\[(-3ab)^2 = (-3)^2 \cdot a^2 \cdot b^2 = 9a^2b^2\]

\[\frac{-1}{(2x)^3} = \frac{-1}{2^3 \cdot x^3} = \frac{-1}{8x^3}\]
Quotient of Powers Property

\[
\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0
\]

Examples:

\[
\frac{x^6}{x^5} = x^{6-5} = x^1 = x
\]

\[
\frac{y^{-3}}{y^{-5}} = y^{-3} - (-5) = y^2
\]

\[
\frac{a^4}{a^4} = a^{4-4} = a^0 = 1
\]
Power of Quotient Property

\[(\frac{a}{b})^m = \frac{a^m}{b^m}, b \neq 0\]

Examples:

\[\left(\frac{y}{3}\right)^4 = \frac{y^4}{3^4}\]

\[\left(\frac{5}{t}\right)^{-3} = \frac{5^{-3}}{t^{-3}} = \frac{1}{5^3} = \frac{t^3}{5^3} = \frac{t^3}{125}\]
## Polynomial

<table>
<thead>
<tr>
<th>Example</th>
<th>Name</th>
<th>Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>monomial</td>
<td>1 term</td>
</tr>
<tr>
<td>6x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3t - 1$</td>
<td>binomial</td>
<td>2 terms</td>
</tr>
<tr>
<td>$12xy^3 + 5x^4y$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2x^2 + 3x - 7$</td>
<td>trinomial</td>
<td>3 terms</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nonexample</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5m^n - 8$</td>
<td>variable exponent</td>
</tr>
<tr>
<td>$n^{-3} + 9$</td>
<td>negative exponent</td>
</tr>
</tbody>
</table>
Degree of a Polynomial

The largest exponent or the largest sum of exponents of a term within a polynomial

Example:

\[ 6a^3 + 3a^2b^3 - 21 \]

<table>
<thead>
<tr>
<th>Term</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6a^3)</td>
<td>3</td>
</tr>
<tr>
<td>(3a^2b^3)</td>
<td>5</td>
</tr>
<tr>
<td>(-21)</td>
<td>0</td>
</tr>
</tbody>
</table>

Degree of polynomial: 5
Leading Coefficient

The coefficient of the first term of a polynomial written in descending order of exponents

Examples:

\[7a^3 - 2a^2 + 8a - 1\]

\[-3n^3 + 7n^2 - 4n + 10\]

\[16t - 1\]
Add Polynomials

Combine like terms.

Example:

\[(2g^2 + 6g - 4) + (g^2 - g)\]

\[= 2g^2 + 6g - 4 + g^2 - g\]

(Group like terms and add.)

\[= (2g^2 + g^2) + (6g - g) - 4\]

\[= 3g^2 + 5g^2 - 4\]
Add Polynomials

Combine like terms.

Example:

\[(2g^3 + 6g^2 - 4) + (g^3 - g - 3)\]

(Align like terms and add.)

\[
\begin{align*}
2g^3 & + 6g^2 & - 4 \\
+ & g^3 & - g - 3 \\
\hline
3g^3 & + 6g^2 & - g - 7
\end{align*}
\]
Subtract Polynomials

Add the inverse.

Example:

\[(4x^2 + 5) - (-2x^2 + 4x -7)\]

(Add the inverse.)

\[= (4x^2 + 5) + (2x^2 - 4x +7)\]

\[= 4x^2 + 5 + 2x^2 - 4x + 7\]

(Group like terms and add.)

\[= (4x^2 + 2x^2) - 4x + (5 + 7)\]

\[= 6x^2 - 4x + 12\]
Subtract Polynomials

Add the inverse.

Example:

\[(4x^2 + 5) - (-2x^2 + 4x - 7)\]

(Align like terms then add the inverse and add the like terms.)

\[
\begin{align*}
4x^2 &+ 5 \\
\underline{-(2x^2 + 4x - 7)} &\rightarrow \\
6x^2 &- 4x + 12
\end{align*}
\]
Multiply Polynomials

Apply the distributive property.

\[(a + b)(d + e + f)\]

\[(a + b)(d + e + f)\]

\[= a(d + e + f) + b(d + e + f)\]

\[= ad + ae + af + bd + be + bf\]
Multiply Binomials

Apply the distributive property.

\[(a + b)(c + d) =\]
\[a(c + d) + b(c + d) =\]
\[ac + ad + bc + bd\]

Example: \((x + 3)(x + 2)\)

\[= x(x + 2) + 3(x + 2)\]
\[= x^2 + 2x + 3x + 6\]
\[= x^2 + 5x + 6\]
Multiply Binomials

Apply the distributive property.

Example: $(x + 3)(x + 2)$

$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$
 Multiply Binomials

Apply the distributive property.

Example: $(x + 8)(2x - 3)$

$= (x + 8)(2x + -3)$

<table>
<thead>
<tr>
<th></th>
<th>2x^2</th>
<th>-3x</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2x</td>
<td>-3</td>
</tr>
<tr>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8x</td>
<td>-24</td>
</tr>
</tbody>
</table>

$2x^2 + 8x + -3x + -24 = 2x^2 + 5x - 24$
Multiply Binomials: Squaring a Binomial

\[(a + b)^2 = a^2 + 2ab + b^2\]

\[(a - b)^2 = a^2 - 2ab + b^2\]

Examples:

\[(3m + n)^2 = 9m^2 + 2(3m)(n) + n^2\]
\[= 9m^2 + 6mn + n^2\]

\[(y - 5)^2 = y^2 - 2(5)(y) + 25\]
\[= y^2 - 10y + 25\]
Multiply Binomials: Sum and Difference

\[(a + b)(a - b) = a^2 - b^2\]

Examples:

\[(2b + 5)(2b - 5) = 4b^2 - 25\]

\[(7 - w)(7 + w) = 49 + 7w - 7w - w^2\]
\[= 49 - w^2\]
Factors of a Monomial

The number(s) and/or variable(s) that are multiplied together to form a monomial

<table>
<thead>
<tr>
<th>Examples:</th>
<th>Factors</th>
<th>Expanded Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5b^2$</td>
<td>$5 \cdot b^2$</td>
<td>$5 \cdot b \cdot b$</td>
</tr>
<tr>
<td>$6x^2y$</td>
<td>$6 \cdot x^2 \cdot y$</td>
<td>$2 \cdot 3 \cdot x \cdot x \cdot y$</td>
</tr>
<tr>
<td>$-\frac{5p^2q^3}{2}$</td>
<td>$-\frac{5}{2} \cdot p^2 \cdot q^3$</td>
<td>$\frac{1}{2} \cdot (-5) \cdot p \cdot p \cdot q \cdot q \cdot q$</td>
</tr>
</tbody>
</table>
Factoring: Greatest Common Factor

Find the greatest common factor (GCF) of all terms of the polynomial and then apply the distributive property.

Example: \(20a^4 + 8a\)

\[
\begin{align*}
\text{common factors} & \quad 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot a + 2 \cdot 2 \cdot 2 \cdot a \\
\text{GCF} & = 2 \cdot 2 \cdot a = 4a \\
20a^4 + 8a & = 4a(5a^3 + 2)
\end{align*}
\]
Factoring: Perfect Square Trinomials

\[ a^2 + 2ab + b^2 = (a + b)^2 \]
\[ a^2 - 2ab + b^2 = (a - b)^2 \]

Examples:
\[ x^2 + 6x + 9 = x^2 + 2\cdot3\cdot x +3^2 \]
\[ = (x + 3)^2 \]

\[ 4x^2 - 20x + 25 = (2x)^2 - 2\cdot2x\cdot5 + 5^2 \]
\[ = (2x - 5)^2 \]
Factoring: Difference of Two Squares

\[ a^2 - b^2 = (a + b)(a - b) \]

Examples:

\[ x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7) \]

\[ 4 - n^2 = 2^2 - n^2 = (2 - n)(2 + n) \]

\[ 9x^2 - 25y^2 = (3x)^2 - (5y)^2 \]
\[ = (3x + 5y)(3x - 5y) \]
Factoring: Sum and Difference of Cubes

\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

Examples:

\[ 27y^3 + 1 = (3y)^3 + (1)^3 \]
\[ = (3y + 1)(9y^2 - 3y + 1) \]

\[ x^3 - 64 = x^3 - 4^3 = (x - 4)(x^2 + 4x + 16) \]
Difference of Squares

\[ a^2 - b^2 = (a + b)(a - b) \]
Divide Polynomials

Divide each term of the dividend by the monomial divisor

Example:

\[(12x^3 - 36x^2 + 16x) \div 4x\]

\[
\begin{align*}
12x^3 - 36x^2 + 16x &= \frac{12x^3}{4x} - \frac{36x^2}{4x} + \frac{16x}{4x} \\
&= 3x^2 - 9x + 4
\end{align*}
\]
Divide Polynomials by Binomials

Factor and simplify

Example:

\[
\frac{7w^2 + 3w - 4}{w + 1}
\]

\[
= \frac{7w^2 + 3w - 4}{w + 1}
\]

\[
= (7w - 4)(w + 1)
\]

\[
= 7w - 4
\]
Prime Polynomial

Cannot be factored into a product of lesser degree polynomial factors

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
</tr>
<tr>
<td>$3t + 9$</td>
</tr>
<tr>
<td>$x^2 + 1$</td>
</tr>
<tr>
<td>$5y^2 - 4y + 3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nonexample</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 - 4$</td>
<td>$(x + 2)(x - 2)$</td>
</tr>
<tr>
<td>$3x^2 - 3x + 6$</td>
<td>$3(x + 1)(x - 2)$</td>
</tr>
<tr>
<td>$x^3$</td>
<td>$x \cdot x^2$</td>
</tr>
</tbody>
</table>
Square Root

\[ \sqrt{x^2} \]

radical symbol

radicand or argument

Simply square root expressions.

Examples:

\[ \sqrt{9x^2} = \sqrt{3^2 \cdot x^2} = \sqrt{(3x)^2} = 3x \]

\[ -\sqrt{(x - 3)^2} = -(x - 3) = -x + 3 \]

Squaring a number and taking a square root are inverse operations.
Cube Root

Simplify cube root expressions.

Examples:

\[ \sqrt[3]{64} = \sqrt[3]{4^3} = 4 \]

\[ \sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3 \]

\[ \sqrt[3]{x^3} = x \]

Cubing a number and taking a cube root are inverse operations.
$n^{th}$ Root

\[ \sqrt[n]{x^m} = x^{\frac{m}{n}} \]

**Examples:**

\[ \sqrt[5]{64} = \sqrt[5]{4^3} = 4^{\frac{3}{5}} \]

\[ \sqrt[6]{729x^9y^6} = 3x^{\frac{3}{2}}y \]
Product Property of Radicals

The square root of a product equals the product of the square roots of the factors.

\[ \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \]

\( a \geq 0 \) and \( b \geq 0 \)

Examples:

\[ \sqrt{4x} = \sqrt{4} \cdot \sqrt{x} = 2\sqrt{x} \]

\[ \sqrt{5a^3} = \sqrt{5} \cdot \sqrt{a^3} = a\sqrt{5a} \]

\[ 3\sqrt{16} = 3\sqrt{8 \cdot 2} = 3\sqrt{8} \cdot 3\sqrt{2} = 2 \cdot 2 \sqrt{2} \]
Quotient Property of Radicals

The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

\[
\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}
\]

\(a \geq 0\) and \(b > 0\)

Example:

\[
\sqrt{\frac{5}{y^2}} = \frac{\sqrt{5}}{\sqrt{y^2}} = \frac{\sqrt{5}}{y}, \quad y \neq 0
\]
Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$.

Example:

$$(x + 3)(x - 4) = 0$$

$$(x + 3) = 0 \text{ or } (x - 4) = 0$$

$$x = -3 \text{ or } x = 4$$

The solutions are -3 and 4, also called roots of the equation.
Solutions or Roots

\[ x^2 + 2x = 3 \]

Solve using the zero product property.

\[ x^2 + 2x - 3 = 0 \]

\[ (x + 3)(x - 1) = 0 \]

\[ x + 3 = 0 \quad \text{or} \quad x - 1 = 0 \]

\[ x = -3 \quad \text{or} \quad x = 1 \]

The solutions or roots of the polynomial equation are -3 and 1.
Zeros

The zeros of a function $f(x)$ are the values of $x$ where the function is equal to zero.

$$f(x) = x^2 + 2x - 3$$

Find $f(x) = 0$.

$$0 = x^2 + 2x - 3$$
$$0 = (x + 3)(x - 1)$$
$$x = -3 \text{ or } x = 1$$

The zeros are -3 and 1 located at (-3,0) and (1,0).

The zeros of a function are also the solutions or roots of the related equation.
**x-Intercepts**

The *x-intercepts* of a graph are located where the graph crosses the x-axis and where \( f(x) = 0 \).

\[
f(x) = x^2 + 2x - 3
\]

0 = (x + 3)(x - 1)
0 = x + 3 or 0 = x - 1
\[x = -3 \text{ or } x = 1\]

The zeros are -3 and 1.

The *x-intercepts* are:
• -3 or (-3,0)
• 1 or (1,0)
Coordinate Plane

ordered pair \((x, y)\)

\(\text{(abscissa, ordinate)}\)
Linear Equation

\[ Ax + By = C \]

(A, B and C are integers; A and B cannot both equal zero.)

Example:

\[-2x + y = -3\]

The graph of the linear equation is a straight line and represents all solutions \((x, y)\) of the equation.
Linear Equation: Standard Form

Ax + By = C

(A, B, and C are integers; A and B cannot both equal zero.)

Examples:

4x + 5y = -24

x − 6y = 9
Literal Equation

A formula or equation which consists primarily of variables

Examples:

\[ ax + b = c \]

\[ A = \frac{1}{2} bh \]

\[ V = lwh \]

\[ F = \frac{9}{5} C + 32 \]

\[ A = \pi r^2 \]
Vertical Line

\[ x = a \]

(Where \( a \) can be any real number)

Example: \( x = -4 \)

Vertical lines have an undefined slope.
Horizontal Line

\[ y = c \]

(where \( c \) can be any real number)

Example:

\[ y = 6 \]

Horizontal lines have a slope of 0.
# Quadratic Equation

\[ ax^2 + bx + c = 0 \]

\( a \neq 0 \)

## Example: \( x^2 - 6x + 8 = 0 \)

<table>
<thead>
<tr>
<th>Solve by factoring</th>
<th>Solve by graphing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - 6x + 8 = 0 )</td>
<td>Graph the related function ( f(x) = x^2 - 6x + 8 ).</td>
</tr>
<tr>
<td>( (x - 2)(x - 4) = 0 )</td>
<td></td>
</tr>
<tr>
<td>( (x - 2) = 0 ) or ( (x - 4) = 0 )</td>
<td></td>
</tr>
<tr>
<td>( x = 2 ) or ( x = 4 )</td>
<td></td>
</tr>
</tbody>
</table>

Solutions to the equation are 2 and 4; the \( x \)-coordinates where the curve crosses the \( x \)-axis.
Quadratic Equation

\[ ax^2 + bx + c = 0 \]

\[ a \neq 0 \]

Example solved by factoring:

<table>
<thead>
<tr>
<th>Quadratic equation</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - 6x + 8 = 0 )</td>
<td>Quadratic equation</td>
</tr>
<tr>
<td>( (x - 2)(x - 4) = 0 )</td>
<td>Factor</td>
</tr>
<tr>
<td>( (x - 2) = 0 ) or ( (x - 4) = 0 )</td>
<td>Set factors equal to 0</td>
</tr>
<tr>
<td>( x = 2 ) or ( x = 4 )</td>
<td>Solve for x</td>
</tr>
</tbody>
</table>

Solutions to the equation are 2 and 4.
Quadratic Equation

\[ ax^2 + bx + c = 0 \]

\[ a \neq 0 \]

Example solved by graphing:

\[ x^2 - 6x + 8 = 0 \]

Graph the related function \( f(x) = x^2 - 6x + 8 \).

Solutions to the equation are the \( x \)-coordinates (2 and 4) of the points where the curve crosses the \( x \)-axis.
# Quadratic Equation: Number of Real Solutions

\[ ax^2 + bx + c = 0, \ a \neq 0 \]

<table>
<thead>
<tr>
<th>Examples</th>
<th>Graphs</th>
<th>Number of Real Solutions/Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - x = 3 )</td>
<td><img src="image" alt="Graph" /></td>
<td>2</td>
</tr>
<tr>
<td>( x^2 + 16 = 8x )</td>
<td><img src="image" alt="Graph" /></td>
<td>1 distinct root with a multiplicity of two</td>
</tr>
<tr>
<td>( 2x^2 - 2x + 3 = 0 )</td>
<td><img src="image" alt="Graph" /></td>
<td>0</td>
</tr>
</tbody>
</table>
Identity Property of Addition

\[ a + 0 = 0 + a = a \]

Examples:

\[ 3.8 + 0 = 3.8 \]
\[ 6x + 0 = 6x \]
\[ 0 + (-7 + r) = -7 + r \]

Zero is the additive identity.
Inverse Property of Addition

\[ a + (-a) = (-a) + a = 0 \]

Examples:

\[ 4 + (-4) = 0 \]
\[ 0 = (-9.5) + 9.5 \]
\[ x + (-x) = 0 \]
\[ 0 = 3y + (-3y) \]
Commutative Property of Addition

\[ a + b = b + a \]

Examples:

\[ 2.76 + 3 = 3 + 2.76 \]
\[ x + 5 = 5 + x \]
\[ (a + 5) - 7 = (5 + a) - 7 \]
\[ 11 + (b - 4) = (b - 4) + 11 \]
Associative Property of Addition

\[(a + b) + c = a + (b + c)\]

Examples:

\[
\left(5 + \frac{3}{5}\right) + \frac{1}{10} = 5 + \left(\frac{3}{5} + \frac{1}{10}\right)
\]

\[3x + (2x + 6y) = (3x + 2x) + 6y\]
Identity Property of Multiplication

\[ a \cdot 1 = 1 \cdot a = a \]

Examples:

\[
3.8 \cdot 1 = 3.8
\]

\[
6x \cdot 1 = 6x
\]

\[
1(-7) = -7
\]

One is the multiplicative identity.
Inverse Property of Multiplication

\[ a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \]

\[ a \neq 0 \]

Examples:

\[ 7 \cdot \frac{1}{7} = 1 \]
\[ \frac{5}{x} \cdot \frac{x}{5} = 1, \ x \neq 0 \]
\[ \frac{-1}{3} \cdot (-3p) = 1p = p \]

The multiplicative inverse of \( a \) is \( \frac{1}{a} \).
Commutative Property of Multiplication

$$ab = ba$$

Examples:

$$(-8) \left( \frac{2}{3} \right) = \left( \frac{2}{3} \right) (-8)$$

$$y \cdot 9 = 9 \cdot y$$

$$4(2x \cdot 3) = 4(3 \cdot 2x)$$

$$8 + 5x = 8 + x \cdot 5$$
Associative Property of Multiplication

\[(ab)c = a(bc)\]

Examples:

\[(1 \cdot 8) \cdot 3\frac{3}{4} = 1 \cdot (8 \cdot 3\frac{3}{4})\]

\[(3x)x = 3(x \cdot x)\]
Distributive Property

\[ a(b + c) = ab + ac \]

Examples:

\[ 5 \left( y - \frac{1}{3} \right) = (5 \cdot y) - \left( 5 \cdot \frac{1}{3} \right) \]

\[ 2 \cdot x + 2 \cdot 5 = 2(x + 5) \]

\[ 3.1a + (1)(a) = (3.1 + 1)a \]
Distributive Property

\[ 4(y + 2) = 4y + 4(2) \]
Multiplicative Property of Zero

\[ a \cdot 0 = 0 \text{ or } 0 \cdot a = 0 \]

Examples:

\[ \frac{2}{3} \cdot 0 = 0 \]

\[ 0 \cdot (-13y - 4) = 0 \]
Substitution Property

If \( a = b \), then \( b \) can replace \( a \) in a given equation or inequality.

Examples:

<table>
<thead>
<tr>
<th>Given</th>
<th>Given</th>
<th>Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 9 )</td>
<td>( 3r = 27 )</td>
<td>( 3(9) = 27 )</td>
</tr>
<tr>
<td>( b = 5a )</td>
<td>( 24 &lt; b + 8 )</td>
<td>( 24 &lt; 5a + 8 )</td>
</tr>
<tr>
<td>( y = 2x + 1 )</td>
<td>( 2y = 3x - 2 )</td>
<td>( 2(2x + 1) = 3x - 2 )</td>
</tr>
</tbody>
</table>
Reflexive Property of Equality

\[ a = a \]

\( a \) is any real number

Examples:

\[ -4 = -4 \]

\[ 3.4 = 3.4 \]

\[ 9y = 9y \]
Symmetric Property of Equality

If \( a = b \), then \( b = a \).

Examples:

If \( 12 = r \), then \( r = 12 \).

If \( -14 = z + 9 \), then \( z + 9 = -14 \).

If \( 2.7 + y = x \), then \( x = 2.7 + y \).
Transitive Property of Equality

If $a = b$ and $b = c$, then $a = c$.

Examples:

If $4x = 2y$ and $2y = 16$, then $4x = 16$.

If $x = y - 1$ and $y - 1 = -3$, then $x = -3$. 
Inequality
An algebraic sentence comparing two quantities

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
<td>less than</td>
</tr>
<tr>
<td>≤</td>
<td>less than or equal to</td>
</tr>
<tr>
<td>&gt;</td>
<td>greater than</td>
</tr>
<tr>
<td>≥</td>
<td>greater than or equal to</td>
</tr>
<tr>
<td>≠</td>
<td>not equal to</td>
</tr>
</tbody>
</table>

Examples:

-10.5 > -9.9 – 1.2
8 > 3t + 2
x – 5y ≥ -12
r ≠ 3
# Graph of an Inequality

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Examples</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; or &gt;</td>
<td>x &lt; 3</td>
<td><img src="image" alt="Graph of x &lt; 3" /></td>
</tr>
<tr>
<td>≤ or ≥</td>
<td>-3 ≥ y</td>
<td><img src="image" alt="Graph of -3 ≥ y" /></td>
</tr>
<tr>
<td>≠</td>
<td>t ≠ -2</td>
<td><img src="image" alt="Graph of t ≠ -2" /></td>
</tr>
</tbody>
</table>
Transitive Property of Inequality

<table>
<thead>
<tr>
<th>If</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a &lt; b ) and ( b &lt; c )</td>
<td>( a &lt; c )</td>
</tr>
<tr>
<td>( a &gt; b ) and ( b &gt; c )</td>
<td>( a &gt; c )</td>
</tr>
</tbody>
</table>

Examples:

- If \( 4x < 2y \) and \( 2y < 16 \), then \( 4x < 16 \).
- If \( x > y - 1 \) and \( y - 1 > 3 \), then \( x > 3 \).
Addition/Subtraction Property of Inequality

<table>
<thead>
<tr>
<th>If</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a &gt; b$</td>
<td>$a + c &gt; b + c$</td>
</tr>
<tr>
<td>$a \geq b$</td>
<td>$a + c \geq b + c$</td>
</tr>
<tr>
<td>$a &lt; b$</td>
<td>$a + c &lt; b + c$</td>
</tr>
<tr>
<td>$a \leq b$</td>
<td>$a + c \leq b + c$</td>
</tr>
</tbody>
</table>

Example:

\[
d - 1.9 \geq -8.7
\]
\[
d - 1.9 + 1.9 \geq -8.7 + 1.9
\]
\[
d \geq -6.8
\]
### Multiplication Property of Inequality

<table>
<thead>
<tr>
<th>If</th>
<th>Case</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a &lt; b$</td>
<td>$c &gt; 0$, positive</td>
<td>$ac &lt; bc$</td>
</tr>
<tr>
<td>$a &gt; b$</td>
<td>$c &gt; 0$, positive</td>
<td>$ac &gt; bc$</td>
</tr>
<tr>
<td>$a &lt; b$</td>
<td>$c &lt; 0$, negative</td>
<td>$ac &gt; bc$</td>
</tr>
<tr>
<td>$a &gt; b$</td>
<td>$c &lt; 0$, negative</td>
<td>$ac &lt; bc$</td>
</tr>
</tbody>
</table>

**Example:** if $c = -2$

- $5 > -3$
- $5(-2) < -3(-2)$
- $-10 < 6$
## Division Property of Inequality

<table>
<thead>
<tr>
<th>If</th>
<th>Case</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a &lt; b$</td>
<td>$c &gt; 0, \text{ positive}$</td>
<td>$\frac{a}{c} &lt; \frac{b}{c}$</td>
</tr>
<tr>
<td>$a &gt; b$</td>
<td>$c &gt; 0, \text{ positive}$</td>
<td>$\frac{a}{c} &gt; \frac{b}{c}$</td>
</tr>
<tr>
<td>$a &lt; b$</td>
<td>$c &lt; 0, \text{ negative}$</td>
<td>$\frac{a}{c} &gt; \frac{b}{c}$</td>
</tr>
<tr>
<td>$a &gt; b$</td>
<td>$c &lt; 0, \text{ negative}$</td>
<td>$\frac{a}{c} &lt; \frac{b}{c}$</td>
</tr>
</tbody>
</table>

**Example:** if $c = -4$

\[\begin{align*}
-90 & \geq -4t \\
\frac{-90}{-4} & \leq \frac{-4t}{-4} \\
22.5 & \leq t
\end{align*}\]
Linear Equation: Slope-Intercept Form

\[ y = mx + b \]

(slope is \( m \) and y-intercept is \( b \))

Example: \( y = \frac{-4}{3} x + 5 \)

\[ m = \frac{-4}{3} \]

\[ b = 5 \]
Linear Equation: Point-Slope Form

\[ y - y_1 = m(x - x_1) \]
where \( m \) is the slope and \((x_1, y_1)\) is the point

Example:
Write an equation for the line that passes through the point \((-4,1)\) and has a slope of 2.

\[
\begin{align*}
  y - 1 &= 2(x - (-4)) \\
  y - 1 &= 2(x + 4) \\
  y &= 2x + 9
\end{align*}
\]
Slope

A number that represents the rate of change in y for a unit change in x.

\[ \text{Slope} = \frac{2}{3} \]

The slope indicates the steepness of a line.
Slope Formula

The ratio of vertical change to horizontal change

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$
Slopes of Lines

Line $p$ has a positive slope.

Line $n$ has a negative slope.

Vertical line $s$ has an undefined slope.

Horizontal line $t$ has a zero slope.
Perpendicular Lines

Lines that intersect to form a right angle

Perpendicular lines (not parallel to either of the axes) have slopes whose product is -1.

Example:

The slope of line $n = -2$. The slope of line $p = \frac{1}{2}$.

$-2 \cdot \frac{1}{2} = -1$, therefore, $n$ is perpendicular to $p$. 
Parallel Lines

Lines in the same plane that do not intersect are parallel. Parallel lines have the same slopes.

Example:

The slope of line $a = -2$.
The slope of line $b = -2$.
$-2 = -2$, therefore, $a$ is parallel to $b$. 
Mathematical Notation

<table>
<thead>
<tr>
<th>Set Builder Notation</th>
<th>Read</th>
<th>Other Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>{x \mid 0 &lt; x \leq 3}</td>
<td>The set of all ( x ) such that ( x ) is greater than or equal to 0 and ( x ) is less than 3.</td>
<td>( 0 &lt; x \leq 3 ) (0, 3]</td>
</tr>
<tr>
<td>{y : y \geq -5}</td>
<td>The set of all ( y ) such that ( y ) is greater than or equal to -5.</td>
<td>( y \geq -5 ) [-5, ( \infty )</td>
</tr>
</tbody>
</table>
System of Linear Equations

Solve by graphing:

\[
\begin{align*}
-x + 2y &= 3 \\
2x + y &= 4
\end{align*}
\]

The solution, \((1, 2)\), is the only ordered pair that satisfies both equations (the point of intersection).
System of Linear Equations

Solve by substitution:

\[
\begin{align*}
    x + 4y &= 17 \\
    y &= x - 2
\end{align*}
\]

Substitute \( x - 2 \) for \( y \) in the first equation.

\[
    x + 4(x - 2) = 17
\]

\[
    x = 5
\]

Now substitute 5 for \( x \) in the second equation.

\[
    y = 5 - 2
\]

\[
    y = 3
\]

The solution to the linear system is (5, 3), the ordered pair that satisfies both equations.
Solve by elimination:

\[
\begin{align*}
-5x - 6y &= 8 \\
5x + 2y &= 4
\end{align*}
\]

Add or subtract the equations to eliminate one variable.

\[
\begin{align*}
-5x - 6y &= 8 \\
+ 5x + 2y &= 4 \\
\hline
-4y &= 12 \\
y &= -3
\end{align*}
\]

Now substitute -3 for y in either original equation to find the value of x, the eliminated variable.

\[
-5x - 6(-3) = 8 \\
x = 2
\]

The solution to the linear system is (2, -3), the ordered pair that satisfies both equations.
## System of Linear Equations

### Identifying the Number of Solutions

<table>
<thead>
<tr>
<th>Number of Solutions</th>
<th>Slopes and y-intercepts</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>One solution</td>
<td>Different slopes</td>
<td><img src="image1.png" alt="Graph" /></td>
</tr>
<tr>
<td>No solution</td>
<td>Same slope and different y-intercepts</td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Infinitely many solutions</td>
<td>Same slope and same y-intercepts</td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
The solutions, (-1,0) and (2,3), are the only ordered pairs that satisfy both equations (the points of intersection).
Graphing Linear Inequalities

<table>
<thead>
<tr>
<th>Example</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y \leq x + 2$</td>
<td><img src="image1.png" alt="Graph" /></td>
</tr>
<tr>
<td>$y &gt; -x - 1$</td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
System of Linear Inequalities

Solve by graphing:

\[
\begin{align*}
  y &> x - 3 \\
  y &\leq -2x + 3
\end{align*}
\]

The solution region contains all ordered pairs that are solutions to both inequalities in the system.

\((-1,1)\) is one solution to the system located in the solution region.
Dependent and Independent Variable

\[ x, \text{ independent variable} \]

(input values or domain set)

Example:

\[ y = 2x + 7 \]

\[ y, \text{ dependent variable} \]

(output values or range set)
Dependent and Independent Variable

Determine the distance a car will travel going 55 mph.

\[ d = 55h \]

<table>
<thead>
<tr>
<th>independent ( h )</th>
<th>dependent ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>165</td>
</tr>
</tbody>
</table>
Graph of a Quadratic Equation

\[ y = ax^2 + bx + c \]

\( a \neq 0 \)

Example:

\[ y = x^2 + 2x - 3 \]

The graph of the quadratic equation is a curve (parabola) with one line of symmetry and one vertex.
Quadratic Formula

Used to find the solutions to any quadratic equation of the form, $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Relations

Representations of relationships

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
</tr>
</tbody>
</table>

Example 1

$\{(0,4), (0,3), (0,2), (0,1)\}$

Example 3
Functions

Representations of functions

Example 1

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

Example 3

{$(-3,4), (0,3), (1,2), (4,6)$}

Example 4
Function

A relationship between two quantities in which every input corresponds to exactly one output.

A relation is a function if and only if each element in the domain is paired with a unique element of the range.
Domain

A set of input values of a relation

Examples:

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$g(x)$</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The **domain** of $g(x)$ is \{-2, -1, 0, 1\}.

The **domain** of $f(x)$ is all real numbers.
Range

A set of output values of a relation

Examples:

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$g(x)$</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The range of $g(x)$ is $\{0, 1, 2, 3\}$.

The range of $f(x)$ is all real numbers greater than or equal to zero.
Function Notation

\[ f(x) \]

\( f(x) \) is read “the value of \( f \) at \( x \)” or “\( f \) of \( x \)”

Example:
\[ f(x) = -3x + 5, \text{ find } f(2). \]
\[ f(2) = -3(2) + 5 \]
\[ f(2) = -6 \]

Letters other than \( f \) can be used to name functions, e.g., \( g(x) \) and \( h(x) \)
Parent Functions

Linear
\[ f(x) = x \]

Quadratic
\[ f(x) = x^2 \]
Parent Functions

**Absolute Value**

\[ f(x) = |x| \]

**Square Root**

\[ f(x) = \sqrt{x} \]
Parent Functions

Cubic

\[ f(x) = x^3 \]

Cube Root

\[ f(x) = \sqrt[3]{x} \]
Parent Functions

Rational

\[ f(x) = \frac{1}{x} \]

Rational

\[ f(x) = \frac{1}{x^2} \]
Parent Functions

Exponential

\[ f(x) = b^x \]

\[ b > 1 \]

Logarithmic

\[ f(x) = \log_b x \]

\[ b > 1 \]
# Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

<table>
<thead>
<tr>
<th>Translations</th>
<th>Description</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x) = f(x) + k$</td>
<td>is the graph of $f(x)$ translated vertically --</td>
<td>$k$ units <strong>up</strong> when $k &gt; 0$.</td>
</tr>
<tr>
<td>$g(x) = f(x - h)$</td>
<td>is the graph of $f(x)$ translated horizontally --</td>
<td>$h$ units <strong>right</strong> when $h &gt; 0$.</td>
</tr>
</tbody>
</table>

$k$ units **down** when $k < 0$. $h$ units **left** when $h < 0$. |
Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

<table>
<thead>
<tr>
<th>Reflections</th>
<th>$g(x) = -f(x)$ is the graph of $f(x)$ – reflected over the $x$-axis.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x) = f(-x)$ is the graph of $f(x)$ – reflected over the $y$-axis.</td>
<td></td>
</tr>
</tbody>
</table>
Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

<table>
<thead>
<tr>
<th>Dilations</th>
<th>$g(x) = a \cdot f(x)$ is the graph of $f(x)$ –</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>vertical dilation</strong> (stretch) if $a &gt; 1$.</td>
</tr>
<tr>
<td></td>
<td><strong>vertical dilation</strong> (compression) if $0 &lt; a &lt; 1$.</td>
</tr>
<tr>
<td>$g(x) = f(ax)$</td>
<td><strong>horizontal dilation</strong> (compression) if $a &gt; 1$.</td>
</tr>
<tr>
<td></td>
<td><strong>horizontal dilation</strong> (stretch) if $0 &lt; a &lt; 1$.</td>
</tr>
</tbody>
</table>
Transformational Graphing

Linear functions

\[ g(x) = x + b \]

Examples:

\[ f(x) = x \]
\[ t(x) = x + 4 \]
\[ h(x) = x - 2 \]

Vertical translation of the parent function,

\[ f(x) = x \]
Transformational Graphing

Linear functions

\[ g(x) = mx \]

\( m > 0 \)

Examples:

\[ f(x) = x \]
\[ t(x) = 2x \]
\[ h(x) = \frac{1}{2}x \]

Vertical dilation (stretch or compression) of the parent function, \( f(x) = x \)
Transformational Graphing

Linear functions

\[ g(x) = mx \]

\[ m < 0 \]

Examples:

\[ f(x) = x \]
\[ t(x) = -x \]
\[ h(x) = -3x \]
\[ d(x) = -\frac{1}{3}x \]

Vertical dilation (\textit{stretch} or \textit{compression}) with a reflection of \( f(x) = x \)
Transformational Graphing

Quadratic functions

\[ h(x) = x^2 + c \]

Examples:

\[ f(x) = x^2 \]
\[ g(x) = x^2 + 2 \]
\[ t(x) = x^2 - 3 \]

Vertical translation of \( f(x) = x^2 \)
Transformational Graphing

Quadratic functions

\[ h(x) = ax^2 \]

\( a > 0 \)

Examples:

\[ f(x) = x^2 \]
\[ g(x) = 2x^2 \]
\[ t(x) = \frac{1}{3}x^2 \]

Vertical dilation (stretch or compression) of \( f(x) = x^2 \)
Transformational Graphing

Quadratic functions

Vertical dilation (stretch or compression) with a reflection of $f(x) = x^2$

Examples:

- $g(x) = -2x^2$
- $t(x) = -\frac{1}{3}x^2$
- $f(x) = x^2$

$a > 0$

Graphs
Transformational Graphing

Quadratic functions

\( h(x) = (x + c)^2 \)

Examples:

\( f(x) = x^2 \)
\( g(x) = (x + 2)^2 \)
\( t(x) = (x - 3)^2 \)

Horizontal translation of \( f(x) = x^2 \)
Inverse of a Function

The graph of an inverse function is the reflection of the original graph over the line, \( y = x \).

Example:

\[ f(x) = \sqrt{x} \]

Domain is restricted to \( x \geq 0 \).

\[ f^{-1}(x) = x^2 \]

Domain is restricted to \( x \geq 0 \).

Restrictions on the domain may be necessary to ensure the inverse relation is also a function.
Discontinuity
Vertical and Horizontal Asymptotes

Example:
\[ f(x) = \frac{1}{x+2} \]

\( f(-2) \) is not defined, so \( f(x) \) is discontinuous.
Discontinuity

Removable Discontinuity
Point Discontinuity

Example:

\[ f(x) = \frac{x^2 + x - 6}{x - 2} \]

\( f(2) \) is not defined.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>error</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

\[ f(x) = \frac{x^2 + x - 6}{x - 2} \]
\[ = \frac{(x + 3)(x - 2)}{x - 2} \]
\[ = x + 3, \ x \neq 2 \]
Direct Variation

\[ y = kx \text{ or } k = \frac{y}{x} \]

constant of variation, \( k \neq 0 \)

Example:
\[ y = 3x \text{ or } 3 = \frac{y}{x} \]

The graph of all points describing a direct variation is a line passing through the origin.
Inverse Variation

\[ y = \frac{k}{x} \quad \text{or} \quad k = xy \]

constant of variation, \( k \neq 0 \)

Example:

\[ y = \frac{3}{x} \quad \text{or} \quad xy = 3 \]

The graph of all points describing an inverse variation relationship are 2 curves that are reflections of each other.
Joint Variation

\[ z = kxy \quad \text{or} \quad k = \frac{z}{xy} \]

constant of variation, \( k \neq 0 \)

Examples:
Area of a triangle varies jointly as its length of the base, \( b \), and its height, \( h \).

\[ A = \frac{1}{2}bh \]

For Company ABC, the shipping cost in dollars, \( C \), for a package varies jointly as its weight, \( w \), and size, \( s \).

\[ C = 2.47ws \]
Arithmetic Sequence

A sequence of numbers that has a common difference between every two consecutive terms

Example: -4, 1, 6, 11, 16 ...

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

The common difference is the slope of the line of best fit.
Geometric Sequence

A sequence of numbers in which each term after the first term is obtained by multiplying the previous term by a constant ratio.

Example: 4, 2, 1, 0.5, 0.25 ...

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Probability

The likelihood of an event occurring

\[
\text{probability of an event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

Example: What is the probability of drawing an A from the bag of letters shown?

\[
P(A) = \frac{3}{7}
\]
Probability of Independent Events

Example:

What is the probability of landing on green on the first spin and then landing on yellow on the second spin?

\[
P(\text{green and yellow}) = P(\text{green}) \cdot P(\text{yellow}) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}
\]
Probability of Dependent Events

Example:

What is the probability of selecting a red jelly bean on the first pick and without replacing it, selecting a blue jelly bean on the second pick?

\[ P(\text{red and blue}) = P(\text{red}) \cdot P(\text{blue} \mid \text{red}) = \frac{4}{12} \cdot \frac{2}{11} = \frac{8}{132} = \frac{2}{33} \]

“blue after red”
Fundamental Counting Principle

If there are \( m \) ways for one event to occur and \( n \) ways for a second event to occur, then there are \( m \cdot n \) ways for both events to occur.

Example:

How many outfits can Joey make using 3 pairs of pants and 4 shirts?

\[ 3 \cdot 4 = 12 \text{ outfits} \]
Permutation

An ordered arrangement of a group of objects

\[ \text{△} \quad \text{○} \quad \text{□} \quad \text{is different from} \quad \text{○} \quad \text{□} \quad \text{△} \]

Both arrangements are included in possible outcomes.

Example:

5 people to fill 3 chairs (order matters). How many ways can the chairs be filled?

1\textsuperscript{st} chair – 5 people to choose from
2\textsuperscript{nd} chair – 4 people to choose from
3\textsuperscript{rd} chair – 3 people to choose from

# possible arrangements are 5 \cdot 4 \cdot 3 = 60
Permutation

To calculate the number of permutations

$$n^P r = \frac{n!}{(n - r)!}$$

$n$ and $r$ are positive integers, $n \geq r$, and $n$ is the total number of elements in the set and $r$ is the number to be ordered.

Example: There are 30 cars in a car race. The first-, second-, and third-place finishers win a prize. How many different arrangements of the first three positions are possible?

$$30^P 3 = \frac{30!}{(30-3)!} = \frac{30!}{27!} = 24360$$
Combination

The number of possible ways to select or arrange objects when there is no repetition and order does not matter.

Example: If Sam chooses 2 selections from heart, club, spade and diamond. How many different combinations are possible?

Order (position) does not matter so

\[ \heartsuit \spadesuit \] is the same as \[ \clubsuit \diamondsuit \]

There are 6 possible combinations.
Combination

To calculate the number of possible combinations using a formula

\[ n^C_r = \frac{n!}{r!(n-r)!} \]

\( n \) and \( r \) are positive integers, \( n \geq r \), and \( n \) is the total number of elements in the set and \( r \) is the number to be ordered.

Example: In a class of 24 students, how many ways can a group of 4 students be arranged?

\[ 24^C_4 = \frac{24!}{4!(24-4)!} = 10,626 \]
# Statistics Notation

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$i^{\text{th}}$ element in a data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>mean of the data set</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>variance of the data set</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation of the data set</td>
</tr>
<tr>
<td>$n$</td>
<td>number of elements in the data set</td>
</tr>
</tbody>
</table>
Mean

A measure of central tendency

Example:
Find the mean of the given data set.
Data set: 0, 2, 3, 7, 8

Balance Point

Numerical Average

\[ \mu = \frac{0 + 2 + 3 + 7 + 8}{5} = \frac{20}{5} = 4 \]
Median

A measure of central tendency

Examples:
Find the median of the given data sets.

Data set: 6, 7, 8, 9, 9
The median is 8.

Data set: 5, 6, 8, 9, 11, 12
The median is 8.5.
# Mode

A measure of central tendency

## Examples:

<table>
<thead>
<tr>
<th>Data Sets</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 4, 6, 6, 6, 6, 10, 11, 14</td>
<td>6</td>
</tr>
<tr>
<td>0, 3, 4, 5, 6, 7, 9, 10</td>
<td>none</td>
</tr>
<tr>
<td>5.2, 5.2, 5.2, 5.6, 5.8, 5.9, 6.0</td>
<td>5.2</td>
</tr>
<tr>
<td>1, 1, 2, 5, 6, 7, 7, 9, 11, 12</td>
<td>1, 7 bimodal</td>
</tr>
</tbody>
</table>
Box-and-Whisker Plot

A graphical representation of the five-number summary

Interquartile Range (IQR)
Summation

This expression means sum the values of $x$, starting at $x_1$ and ending at $x_n$.

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots + x_n$$

Example: Given the data set \{3, 4, 5, 5, 10, 17\}

$$\sum_{i=1}^{6} x_i = 3 + 4 + 5 + 5 + 10 + 17 = 44$$
Mean Absolute Deviation

A measure of the spread of a data set

\[
\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{n} |x_i - \mu|}{n}
\]

The mean of the sum of the absolute value of the differences between each element and the mean of the data set
Variance

A measure of the spread of a data set

\[
\text{variance}(\sigma^2) = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}
\]

The mean of the squares of the differences between each element and the mean of the data set
Standard Deviation

A measure of the spread of a data set

\[
\text{standard deviation} (\sigma) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}
\]

The square root of the mean of the squares of the differences between each element and the mean of the data set or the square root of the variance
Standard Deviation

A measure of the spread of a data set

\[
\text{standard deviation} (\sigma) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}
\]

Comparison of two distributions with same mean and different standard deviation values
z-Score

The number of standard deviations an element is away from the mean

\[
z\text{-score}(z) = \frac{x - \mu}{\sigma}
\]

where \(x\) is an element of the data set, \(\mu\) is the mean of the data set, and \(\sigma\) is the standard deviation of the data set.

Example: Data set A has a mean of 83 and a standard deviation of 9.74. What is the z-score for the element 91 in data set A?

\[
z = \frac{91 - 83}{9.74} = 0.821
\]
z-Score

The number of standard deviations an element is from the mean

\[ z\text{-score (z)} = \frac{x - \mu}{\sigma} \]
Elements within one standard deviation of the mean

Given $\mu = 45$ and $\sigma = 24$

$Z = 1$

Normal Distribution

approximate percentage of element distribution
Elements within One Standard Deviation ($\sigma$) of the Mean ($\mu$)

$\mu - \sigma$

$z = -1$

$\mu + \sigma$

$z = 1$

$\mu = 12$

$\sigma = 3.49$
Scatterplot

Graphical representation of the relationship between two numerical sets of data

\[ y \]
\[ x \]
Positive Correlation

In general, a relationship where the dependent \((y)\) values increase as independent values \((x)\) increase.
Negative Correlation

In general, a relationship where the dependent \((y)\) values decrease as independent \((x)\) values increase.
No Correlation

No relationship between the dependent \((y)\) values and independent \((x)\) values.
Curve of Best Fit

Calories and Fat Content

\[ y = 11.731x + 193.85 \]

Height of a Shot Put

\[ y = -0.01x^2 + 0.7x + 6 \]
Curve of Best Fit

Height of a Shot Put

Bacteria Growth Over Time

$y = -0.01x^2 + 0.7x + 6$

$y = 20.512 (1.923)^x$
Outlier Data

**Wingspan vs. Height**

![Graph showing the relationship between wingspan and height with an outlier data point highlighted.]

**Gas Mileage for Gasoline-fueled Cars**

![Bar graph showing frequency distribution of gas mileage with an outlier data point highlighted.]